Analyzing Inventory Requirements in a Custom Scoop Order System Using Combinatorics

Livy Chandra - 13525150
Program Studi Teknik Informatika
Sekolah Teknik Elektro dan Informatika
Institut Teknologi Bandung, Jalan Ganesha 10 Bandung
E-mail: ivyychandra@gmail.com , 13525150@std.stei.itb.ac.id

***Abstract*—Custom Scoop Order is a sales system in which customers receive products determined by a random bead-scooping process, while often being allowed to request specific product attributes such as colors and characters. As the number of categories and attributes increases, inventory management becomes more complex due to the growing number of possible fulfillment scenarios. This paper applies combinatorial methods to analyze inventory requirements in a Custom Scoop Order system. The Stars and Bars method and Complement Method are used to analyze scoop outcomes, while the Multiplication Principle and Inclusion-Exclusion Principle are used to examine attribute expansion and substitution strategies. The results show that increasing item categories causes rapid combinatorial growth in the outcome space, whereas increasing attributes affects only the number of product variants. Furthermore, attribute-based substitution increases fulfillment flexibility while reducing inventory pressure. These findings suggest that attribute expansion combined with substitution strategies provides a more scalable approach to increasing product variety than continuous category expansion.**

Keywords—Combinatorics; Custom Scoop Order; Scoop Order System; Inventory Management.

I. INTRODUCTION

Custom Scoop Order systems have become increasingly popular among small businesses, particularly those operating through social media platforms such as TikTok. In this system, products obtained by customers are determined through a random bead-scooping process. Each bead type represents a specific item category, and the composition of beads obtained determines the items received by the customer. The element of surprise created by the random bead combinations makes the system attractive to customers and helps businesses increase engagement. Item categories typically consist of small novelty goods such as stationery, mini mirrors, keychains, character-themed pouches, or plushies — items purchased primarily for their aesthetic appeal and often featuring popular character themes.

In practice, the design of Custom Scoop Order systems varies considerably between sellers. Some sellers fix the number of beads given per scoop, while others let the scoop size vary by mixing beads with different sizes. The number of item categories also differs: smaller sellers may operate without any request option available to keep stocking simpler and cheaper, while larger sellers with more diverse product catalogs may offer a few request options on color and/or character themes. Sellers further differ in how categories are weighted; some give every bead type an equal probability of being drawn, while others adjust the proportion of beads in the container to make premium categories rarer. These differences mean that no single configuration can represent every Custom Scoop Order business, and the inventory complexity faced by one seller may look very different from another's.

For this paper, a representative configuration is chosen for analysis: four item categories, a fixed scoop size of fifteen beads per scoop, and a uniform draw assumption with no restriction on how many beads of a given type may appear together. Four categories are selected to keep the model analytically clean while still capturing the essential structure of a tiered item system; real sellers often operate with more categories, but the combinatorial patterns identified here extend directly to larger configurations. A fixed scoop size of fifteen beads is chosen as a representative quantity consistent with common seller practice, since many sellers standardize the number of beads per scoop regardless of price tier. It is also worth noting that many business owners who do not formally calculate inventory complexity still manage to operate by intuitively substituting one item variant for another within the same category, a practice this paper formalizes later on. By isolating the core bead-category structure from attribute-level variation, this baseline model can later be extended in Section V.A to incorporate color and character request options, allowing the two sources of inventory complexity, category growth and attribute growth, to be analyzed and compared separately rather than conflated into a single model.

This paper assumes that each scoop outcome is fulfilled exactly as drawn, without partial fulfillment or substitution, as a baseline to measure the full scope of inventory complexity before mitigation strategies are introduced. Different bead compositions produce different item outcomes, requiring businesses to prepare inventory that can satisfy a wide range of customer requests. As the number of item categories increases, the number of possible outcomes also increases, making inventory management more complex.

Combinatorics provides mathematical tools that can be used to analyze the number of possible outcomes generated by a Custom Scoop Order system. Through combinatorial analysis, business owners can estimate the complexity of their inventory requirements and evaluate strategies to simplify fulfillment. This paper applies combinatorial methods to analyze possible scoop outcomes, examine the effect of increasing item categories versus increasing product attributes, and discuss attribute-based substitution as a practical inventory management strategy.

II. Combinatorics Theory

Combinatorics [1] is a branch of mathematics that studies methods of counting and arranging objects. It is commonly used to determine the number of possible outcomes in situations involving multiple choices or constraints. Four combinatorial methods are used throughout this paper.

A. Stars and Bars Method

The Stars and Bars method [2] is used to solve a frequently occurring problem in combinatorics, where the goal is to count the number of ways to group identical objects, such as placing indistinguishable balls into labelled urns. For an equation of the form

$$x_1 + x_2 + ... + x_n = r$$

The number of nonnegative integer solutions is given by

$$C(r+n-1, n-1)$$

This method is used throughout this study to calculate the number of possible scoop compositions.

B. Complement Method

The complement method [3] is a counting technique used when it is easier to count the outcomes that do not satisfy a condition than to count the outcomes that do. Given a universal set U of all possible outcomes and a subset A representing outcomes that satisfy a specific condition, the size of A can be calculated as

$$|A| = |U| - |A^c|$$

where A^c is the complement of A, that is, the set of every outcome that do not satisfy the condition. This method is used in Section IV.B and IV.C because counting outcomes that contain at least one bead of a given type directly would require summing over many valid ranges; counting the complement can reduces to a single Stars and Bars computation, making the approach considerably more efficient.

C. Multiplication Principle (Rule of Product)

The multiplication principle [1] states that if a first task can be performed in p ways, and a second, independent task can be performed in q ways, then the two tasks together can be performed in $p \times q$ ways.

This principle is used in Section IV.G to model the growth of product variants when attribute options such as color and character are added within an existing item category.

D. Inclusion-Exclusion Principle

The Inclusion-Exclusion Principle [1] is a counting technique used to determine the size of the union of overlapping sets. When two sets A and B are counted separately, elements belonging to both sets are counted twice. To correct this overcounting, the intersection must be subtracted once.

For two sets, the principle is given by

$$|A \cup B| = |A| + |B| - |A \cap B|$$

This method is used in Section V.B to determine the number of acceptable product variants under attribute-based substitution policies. Specifically, it is applied when a customer request may be satisfied by preserving either a requested character or a requested color, causing some product variants to belong to both acceptable sets simultaneously.

III. Scoop-System Modeling

A. Assumptions

The analysis is conducted using the following assumptions:

1) *There are four item categories:*
   a) *Sticker*
   b) *Notebook*
   c) *Pouch*
   d) *Bag*
2) *Each category is represented by one bead type.*
3) *Each customer receives exactly fifteen beads per scoop.*
4) *The order of beads does not matter.*
5) *Some bead type may appear multiple times within the same scoop.*
6) *Every valid composition of fifteen beads is considered a possible outcome.*
7) *Each scoop outcome is assumed to be fulfilled exactly as drawn, without partial fulfillment or substitution, unless stated otherwise.*

B. Mathematical Model

Let
x_1 = number of Sticker beads
x_2 = number of Notebook beads
x_3 = number of Pouch beads
x_4 = number of Bag beads

Since each scoop contains exactly fifteen beads, the amount of each bead type on every scoop can be represented with this equation:

$$x_1 + x_2 + x_3 + x_4 = 15$$

Where $x_1, x_2, x_3, x_4 \geq 0$.

The Stars and Bars method is used to determine the number of valid solutions and analyze several categories of outcomes.

IV. Outcome Analysis

A. Total Possible Outcomes

The total number of possible scoop compositions is determined by calculating the number of nonnegative integer solutions to

$$x_1 + x_2 + x_3 + x_4 = 15$$

Using the Stars and Bars method,

$$C(15+4-1, 4-1) = C(18,3) = 816$$

Therefore, there are 816 possible compositions of a fifteen-bead scoop. To illustrate how different these outcomes can be, Table I presents five contrasting scenarios rather than an arbitrary sequence of compositions.

TABLE I. CONTRASTING SCOOP OUTCOME SCENARIOS

Scenario	Beads Distribution			
	x_1 (Sticker)	x_2 (Notebook)	x_3 (Pouch)	x_4 (Bag)
Sticker-only[a]	15	0	0	0
Bag-only[b]	0	0	0	15
Balanced[c]	4	4	4	3
Mixed Premium[d]	8	4	2	1
No Premium[e]	8	7	0	0

a. All beads are the lowest tier item

b. All beads are the highest tier item

c. Roughly equal spread across all tier

d. Majority low-tier with some premium

e. All beads are in non-premium categories (two lowest tier)

These five scenarios were chosen deliberately to span the outcome space, from the least valuable composition a customer could receive to the most valuable, illustrating the scale of variation a business must be prepared to fulfill, rather than simply enumerating five arbitrary solutions.

Although the system contains only four item categories, it can already generate hundreds of unique outcomes. This indicates that inventory planning must accommodate substantial variation in customer rewards.

B. Outcomes Containing at Least One Bag (Scenario D)

Suppose Bags are considered a premium item category. The number of outcomes containing at least one Bag can be determined using the complement method.

First, consider outcomes containing no Bag:

$$x_1 + x_2 + x_3 = 15$$

Using Stars and Bars,

$$C(17,2) = 136$$

Therefore,

Outcomes containing at least one Bag = 816 − 136 = 680

This means that 680 possible scoop compositions contain at least one Bag reward.

C. Outcomes Containing at Least One Bag and One Pouch (Scenario C)

Next, consider outcomes containing both premium item categories. The constraints are $x_3 \geq 1$ and $x_4 \geq 1$, since this scenario specifically targets the subset of outcomes where a customer receives both premium rewards in the same scoop, representing the combined high-value case relevant to premium-inventory planning.

Let $y_3 = x_3 - 1$ and $y_4 = x_4 - 1$. Substituting them into the original equation gives

$$x_1 + x_2 + y_3 + y_4 = 13$$

Using Stars and Bars,

$$C(16,3) = 560$$

Therefore, there are 560 possible outcomes containing both a Bag and a Pouch item. This result shows that many potential customer outcomes involve multiple premium item categories simultaneously.

D. Outcomes Without Premium Rewards (Scenario E)

Suppose the premium categories consist of Bags and Pouches. To determine outcomes containing neither premium category:

$$x_1 + x_2 = 15$$

Using Stars and Bars,

$$C(16,1) = 16$$

Therefore, only 16 possible scoop compositions contain neither Bags nor Pouches. Compared with the total of 816 outcomes, this number is relatively small.

E. Summary of Outcome Analysis

The results are summarized in Table II.

TABLE II. SUMMARY OF OUTCOME ANALYSIS RESULT

Outcome Category	***Count***	***Share of Total (%)***
Total possible outcomes	816	100%
Contains at least one Bag	680	83.3%
Contains at leas one Bag and one Pouch	560	68.6%
Contains neither Bag nor Pouch	16	2.0%

This result carries a direct real-life implication for inventory planning. Since Bag rewards appear in roughly 83% of all possible outcomes and both premium categories appear together in roughly 69% of outcomes, Bag and Pouch rewards *cannot* be treated as rare or occasional cases in stock planning. A seller who assumes premium rewards are uncommon edge cases *risks understocking* them and being unable to fulfill the majority of scoops drawn by customers, which can lead to fulfillment delays or forced substitution with lower-value items, and thus damaging customer trust. The combinatorial result therefore directly informs a practical recommendation that premium-category inventory should be sized closer to baseline demand than to a rare-event buffer.

F. *Effect of Increasing the Number of Item Categories*

Business owners may consider introducing additional item categories to increase variety or attract attention. However, doing so also increases inventory complexity.

For example, a scoop containing fifteen beads for each scoop and n total item categories, the number of possible outcomes is

$$C(15+n-1, n-1)$$

TABLE III. POSSIBLE OUTCOMES WITH INCREASING NUMBER OF ITEM CATEGORIES (N)

Number of Categories (n)	***Possible Outcomes***
4	816
5	3,876
6	15,504
7	54,264
8	170,544

The increase is *substantial*. Adding only one category increases the number of outcomes from 816 to 3,876. By the time eight categories are used, the number of possible outcomes exceeds 170,000. This demonstrates that inventory complexity grows much faster than the number of categories themselves. Therefore, introducing new categories should be carefully evaluated against the resulting increase in fulfillment complexity.

V. INVENTORY STRATEGY

A. *Comparison Between Category Expansion and Attribute Expansion*

A seller seeking to offer more variety does not necessarily need to add new item categories. An alternative is to keep the number of categories fixed and instead let customers request product attributes, such as color, character, or both altogether, within each existing category. These two strategies grow inventory complexity through fundamentally different mechanisms.

Category expansion, as shown in Table III, increases the bead-composition outcome space itself, governed by the Stars and Bars formula $C(15+n-1, n-1)$. This growth compounds with the existing 816 baseline compositions and accelerates rapidly as n increases.

On the other hand, attribute expansion uses the multiplication principle. Suppose the number of bead categories n remains fixed at four, but each category now offers c color options and k character options. By the multiplication principle, the number of distinct product variants required per category becomes $c \times k$, and the total number of distinct stock-keeping units (SKUs, where each SKU represents a unique product variant requiring separate inventory tracking) across all categories becomes

$$n \times c \times k$$

Crucially, this growth path does not affect the scoop-composition outcome space at all, the number of possible scoop outcomes remains

$$C(18,3) = 816$$

regardless of how many color or character options exist, since *beads are still drawn from only four categories*. Attributes only determine which specific product variant fulfills a given category's reward, not how many beads of that category are drawn.

For example, suppose a seller adds 3 color options and 2 character options to each of the 4 categories. By the multiplication principle, the total number of SKUs required is

$$4 \times 3 \times 2 = 24$$

while the scoop-outcome space remains unchanged at 816. Compare this with adding two new bead categories instead (n: 4 → 6, no attributes), the outcome space grows from 816 to 15,504, an increase of roughly 19 times, for a comparatively smaller nominal increase in variety.

TABLE IV. PRODUCT COMBINATIONS WITH INCREASING ATTRIBUTE OPTIONS (N = 4)

Color Options (c)	***Character Options (k)***	***Total SKUs ($n \times c \times k$)***	***Scoop Outcome Space***
1	1	4	816
2	1	8	816
3	2	24	816
4	3	48	816
5	4	80	816

Ranking these two strategies by inventory manageability, *attribute expansion is the more controllable approach*. It produces a small, fully predictable, multiplicative increase in SKU count without altering the combinatorial outcome space that drives scoop-level fulfillment complexity. Category expansion, in contrast, directly inflates the outcome space combinatorially and compounds with the existing baseline, producing far more fulfillment scenarios for a comparatively smaller nominal increase in variety. For sellers whose primary goal is to add perceived novelty without overwhelming inventory planning, increasing attribute options such as color and character is therefore the better strategy, particularly when combined with the attribute-based substitution approach discussed in the following section.

B. *Attribute-Substitution Strategy*

Although attribute expansion is generally more manageable than category expansion, inventory challenges still arise when customers request specific combinations of attributes. In practice, many Custom Scoop Order sellers allow customers to request particular colors, characters, or both. However, fulfilling every request exactly may still require maintaining stock for a large number of product variants.

Suppose a category offers c color options and k character options. By the multiplication principle, the total number of distinct variants within that category is $c \times k$.

For example, if a seller offers four colors and four characters, then $4 \times 4 = 16$ distinct product variants must be managed. Consider a customer requesting a Pink Kuromi Sticker. Under an exact-fulfillment policy, only one of the sixteen variants satisfies the request. Therefore, the number of acceptable variants is

$$F = 1$$

If the seller instead adopts a character-preserving substitution policy, any Kuromi Sticker is considered acceptable regardless of color. Since there are c possible colors, the number of acceptable variants becomes

$$F = c$$

Similarly, under a color-preserving substitution policy, any Pink Sticker is acceptable regardless of character, giving

$$F = k$$

A more flexible strategy is to preserve either the requested character or the requested color. In this case, all variants sharing the requested character and all variants sharing the requested color are acceptable. Applying the Inclusion-Exclusion Principle described in Section II.D, the number of acceptable variants becomes

$$F = c + k - 1$$

because the originally requested variant belongs to both groups and would otherwise be counted twice.

For example, when $c = 4$ and $k = 4$:

TABLE V. ACCEPTABLE VARIANTS UNDER DIFFERENT FULFILLMENT POLICIES

Fulfillment Policy	***Acceptable Variants***
Exact match	1
Same character substitution	4
Same color substitution	4
Same color or character substitution	7

The substitution policy therefore increases the number of acceptable variants from 1 to 7. In other words, the seller gains access to seven times as many inventory options while still partially satisfying the customer's preference.

This result demonstrates mathematically why attribute-based substitution is effective. Rather than requiring stock for every exact attribute combination, the seller can fulfill requests using a broader set of acceptable products. As the number of acceptable variants increases, the likelihood that available inventory can satisfy customer requests also increases, reducing stock pressure and improving fulfillment flexibility.

C. *Practical Inventory Implication*

The analysis in this paper shows that inventory complexity can grow through two fundamentally different mechanisms: category expansion and attribute expansion.

Category expansion increases the scoop-outcome space itself. Because the number of outcomes is governed by the Stars and Bars expression $C(15+n-1, n-1)$, adding new categories causes rapid combinatorial growth in fulfillment scenarios. As shown in Table III, increasing the number of categories from four to eight raises the outcome space from 816 to 170,544 possible compositions.

Attribute expansion behaves differently. Increasing the number of colors or characters increases the number of product variants through the multiplication principle, but *does not affect the scoop-outcome space*. The number of bead compositions remains unchanged because the underlying category structure is unchanged.

Furthermore, attribute-based substitution provides an additional mechanism for controlling inventory complexity. By allowing requests to be fulfilled using products that preserve either the requested color or the requested character, sellers can substantially increase the number of acceptable inventory options without reducing the variety offered to customers.

It should be noted that the simplest inventory-management approach is still to eliminate attribute requests entirely. Under this policy, rewards are assigned solely according to item categories, eliminating the need to maintain inventory for specific color-character combinations and reducing fulfillment complexity to the category level. However, this simplicity comes at the cost of reduced customer customization. Attribute-based substitution therefore serves as a practical compromise, allowing businesses to offer personalized requests while avoiding the full inventory burden associated with exact attribute fulfillment.

Therefore, businesses seeking to offer greater product variety should generally prioritize attribute expansion over category expansion and combine it with attribute-based substitution strategies. This approach provides greater customer choice while avoiding the rapid combinatorial growth associated with adding new item categories.

VI. CONCLUSION

This paper applied combinatorial methods to analyze inventory requirements in a Custom Scoop Order system. Using the Stars and Bars method, it was shown that a scoop containing fifteen beads and four item categories can generate 816 possible outcomes. Additional analysis identified the number of outcomes containing premium reward categories and demonstrated that the number of possible scoop compositions grows rapidly as additional item categories are introduced.

The study further compared category expansion and attribute expansion as two approaches for increasing product variety. Category expansion was shown to increase the scoop-outcome space combinatorially, whereas attribute expansion increases only the number of product variants through the multiplication principle without affecting the underlying outcome space. Consequently, attribute expansion provides a more manageable method for increasing customer choice.

An attribute-based substitution strategy was also analyzed using the Inclusion-Exclusion Principle. The analysis showed that allowing requests to be fulfilled by preserving either a requested color or a requested character increases the number of acceptable product variants from 1 to $c + k - 1$. In the example considered, the number of acceptable variants increased from 1 to 7, demonstrating a substantial improvement in inventory flexibility while still maintaining partial customer preference satisfaction.

Overall, the results reveal a hierarchy of inventory complexity in Custom Scoop Order systems. The simplest approach is to allow no attribute requests, followed by attribute requests with substitution, then exact attribute requests, while continuous category expansion produces the greatest increase in fulfillment complexity. Therefore, businesses seeking to balance customization and inventory manageability should prioritize attribute expansion and attribute-based substitution strategies rather than continuously introducing new item categories.

VII. Limitations

Several simplifying assumptions in this paper warrant acknowledgment. First, the uniform draw assumption that every bead type is equally likely to be selected does not reflect the full range of seller practices. Many sellers deliberately adjust the ratio of beads in the container to make premium-category items rarer, creating a non-uniform probability distribution over the four bead types. Under non-uniform draw probabilities, the expected per-category demand estimate of Section IV.I would shift accordingly, and a more complete model would incorporate the specific bead ratios used by each seller.

Second, the analysis treats all outcomes as equally likely from a fulfillment standpoint, but actual customer demand may be correlated with seasonal trends, character popularity, or promotional events that fall outside the scope of this combinatorial model.

Third, the baseline analysis in Sections IV.A through IV.E assumes exact fulfillment without substitution. While Section IV.H introduces category-based substitution as a mitigation strategy, the full effect of substitution on customer satisfaction and perceived fairness is not formally modeled here and represents a direction for future work.

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PERNYATAAN

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Livy Chandra - 13525150